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COMMENTS ON "THE HIGH-FREQUENCY RESPONSE OF A PLATE CARRYING A CONCENTRATED MASS/SPRING SYSTEM"

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1. INTRODUCTION

Dowel and Tang present an investigation observing the high-frequency response of a plate carrying a concentrated mass/spring system [1]. An understanding of high-frequency dynamics in coupled structures can be expanded through this generic problem. Following closely Kubota *et al.* [2], the authors perform an asymptotic modal analysis (AMA) based upon the assumptions of small damping and a large number of modes in the plate. The results for spatial average vibration in the plate and for the vibration amplitude of the attached oscillator are expressed in closed form. Such simple expressions are of great engineering interest as they allow quick estimates of the severity of vibrations received by small objects mounted on plate-like structures; e.g., delicate electric equipment mounted on a ships hull.

In reference [1], Dowel and Tang make two distressing statements upon which the present author is impelled to comment.

Firstly, in the introduction it is said that AMA is an alternative to statistical energy analysis (SEA) and that: "One of the outstanding unresolved questions for both AMA and SEA is how to treat effectively the dynamics of two or more interconnected systems. The failure of SEA to provide consistently accurate results and, heretofore, the absence of any extension of AMA to such systems has been a major limitation for both methods." Now, this statement is a challenge to anyone involved with SEA as it implies that SEA cannot provide useful information. To rebut this conclusion, a small SEA model of the structure investigated in reference [1] is presented with the results compared to those from an exact calculation, showing good agreement.

Secondly, on p. 848: "Note equation (27a) is the same result obtained in reference [2] for a *rigidly* connected mass". Equation (27a) displays the ratio of mean square (m.s.) vibrations of the mass to the spatial average m.s. vibrations of the plate. This ratio must be affected by the presence of a spring between plate and mass, so some of the results in reference [1] are consequently incorrect; this is demonstrated in what follows.

2. GOVERNING EQUATIONS

The investigated structure consists of a rectangular, thin-walled, plate with dimensions l_x and l_y, which is point excited at position (x_F, y_F) and upon which an

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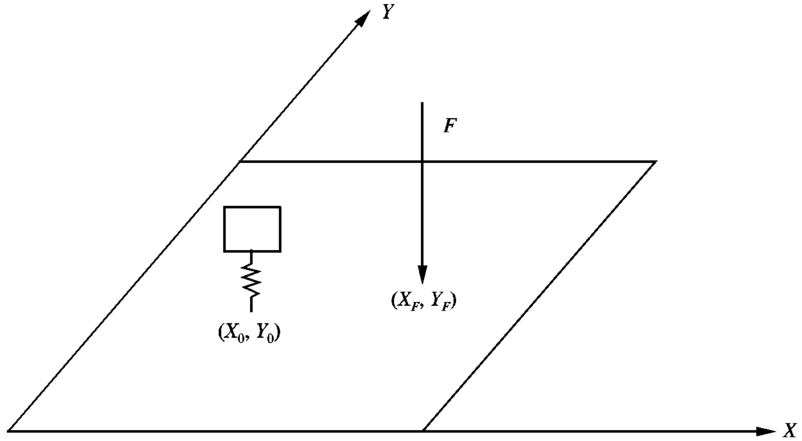

Figure 1. Geometry of the plate structure with concentrated mass/spring system. From reference [1].

oscillator with mass M_o and stiffness K_o is mounted at position (x_o, y_o), see Figure 1. The governing equations of motion are derived in reference [1] but are repeated here for completeness. First, the plate displacement W is expanded in terms of its natural modes ψ_m:

$$W(x, y, \omega) = \sum_m a_m(\omega)\psi_m(x, y), \tag{1}$$

where harmonic motion of the form $e^{i\omega t}$ is assumed. Upon this basis, the coupled equations of motion are given by

$$Z_m a_m = -F\psi_{mF} - f\psi_{mo}, \tag{2}$$

$$-\omega^2 M_o V = f,$$

$$f = \omega_o^2 M_o \sum_m (a_m \psi_{mo}) - \omega_o^2 M_o V,$$

where V is the oscillator mass displacement, f is the spring force, F is the r.m.s. complex amplitude of the applied point force and where

$$\psi_{mF} = \psi_m(x_F, y_F), \qquad \psi_{mo} = \psi_m(x_o, y_o), \tag{3}$$

$$\omega_o^2 = K_o/M_o,$$

$$Z_m = M_m(\omega_m^2 + 2i\zeta_m\omega_m\omega - \omega^2),$$

$$M_m = \Lambda_m M_p, \qquad \Lambda_m A_p = \int_0^{l_x}\int_0^{l_y} \psi_m^2 \mathrm{d}x\mathrm{d}y, \quad M_p = \rho T_p A_p.$$

in which T_p is the plate thickness, A_p is the plate area, ρ is the density, ζ_m is the modal damping ratio and ω_m is the natural frequency associated with the mode ψ_m.

Note, the equations (1–3) are for harmonic motion equal to the corresponding expressions in equations (1–9) in reference [1], with the exceptions that the spring force, f, is explicitly expressed, the dynamic stiffness Z_m is defined for convenience and a slight change in notation.

From equations (2), the spring force is given by

$$f = \frac{-F\sum_m \psi_{mF}\psi_{mo}/Z_m}{\sum_m(\psi_{mo}^2/Z_m) + (1/\omega_o^2 - 1/\omega^2)/M_o}. \tag{4}$$

Upon this basis, the spatial mean square average vibration velocity of the plate and the oscillator velocity are given by

$$w^2 = \sum_m \Lambda_m |\mathrm{i}\omega a_m|^2 = \sum_m \omega^2 \Lambda_m \left| \frac{\mathrm{F}\psi_{mF} + \mathrm{f}\psi_{mo}}{Z_m} \right|^2, \tag{5a}$$

$$v^2 = |\mathrm{i}\omega V|^2 = |f|^2/\omega^2 M_o^2. \tag{5b}$$

Equations (4) and (5) are used to assess the accuracy of the SEA and AMA discussed below. Unless explicitly stated, the data used are those given in Table 1 being the same as those in Section 4.2 of reference [1].

In reference [1], the magnitude of the damping ratio is not specified. However, in the first paragraph on p. 847, it is said that the reduction of a double summation to a single summation "is based on the notion that the resonance peaks of the transfer function are well separated in frequency for small damping". The average separation of resonance is measured by the modal overlap, M, given by

$$M = \eta\omega n_p, \quad \eta = 2\zeta_m, \tag{6}$$

where η is the loss factor and n_p is the modal density, i.e., the average number of modes per unit angular frequency. For a thin-walled plate the modal density is

TABLE 1

Geometrical and material data

l_x	1·2 m	E	72×10^9 N/m^2
l_y	0·8 m	ρ	$2{\cdot}7 \times 10^3$ kg/m^3
T_p	2 mm	ν	0·3
		η	1×10^{-3}
x_F	$2l_x/\pi$	x_o	l_x/π
y_F	$l_y/(2\pi)$	y_o	l_y/π
M_o	0·11 kg	K_o	$\omega_o^2 M_o$ N/m

given by [3]

$$n_p = A_p/3{\cdot}6c_L T_p, \quad c_L = \sqrt{E/\rho(1-\nu^2)}, \tag{7}$$

where E is Young's modulus and ν is the Poisson ratio. Using the values given in Table 1, the modal overlap is $M \approx 0{\cdot}04$ at 1 kHz. The value for the loss factor of $\eta = 1 \times 10^{-3}$ is below that most often found in engineering structures but is used to ensure a fair comparison to the AMA.

3. STATISTICAL ENERGY ANALYSIS

3.1. BACKGROUND

Some 20 years ago, P. W. Smith wrote, "Procedures known as 'statistical energy analysis' (SEA) [4–6] are used to estimate the steady-state dynamical response of complex vibratory systems. They reduce the problem to a set of linear algebraic equations relating energetic variables associated with subsystems of the complete system" [7]. He continued, "Analysis has shown that the SEA are valid in a statistical sense, as relations between average values over an appropriate ensemble of conditions. Most frequently, the average is taken over a frequency band, with the system parameters invariant. Other ensembles may lead to the same result". (This "ergodic" assumption is demonstrated for one-dimensional wave-guide structures approximately a decade later [8,9].)

The starting point in SEA is to write a power balance equation for subsystem i in the form

$$P^i_{in} = P^i_{dis} + \sum_{j \neq i} P^{i,j}_{coup} \tag{8}$$

where P^i_{in} is the input power to the subsystem from external sources, P^i_{dis} is the power dissipated through damping and $P^{i,j}_{coup}$ is the net power transmitted from subsystem i to a neighbouring subsystem j through mechanical coupling. In equation (8), steady state vibration is considered and the powers are time averages. For commonly used damping models the dissipated power is written in the form

$$P^i_{dis} = \eta_i \omega E_i = M_i E_i / n_i \tag{9}$$

where η_i is the damping loss factor (twice the damping ratio), M_i is the modal overlap and E_i is the time-averaged energy stored in the subsystem. Equations (8) and (9) follow from basic physical principles; however, in order to make progress it is necessary to develop an expression of the coupling power, and the form adopted represents the main SEA hypothesis. It is assumed *a priori* that

$$P^{i,j}_{coup} = C^{i,j}\left(\frac{E_i}{n_i} - \frac{E_j}{n_j}\right), \quad C^{i,j} = \eta_c^{i,j}\omega n_i \tag{10}$$

where the non-dimensional number $C^{i,j}$ is the vibration conductivity and $\eta_c^{i,j}$ is the coupling loss factor. Equation (10) states that the net energy flow is proportional to the difference in 'energy per mode' in connected subsystems (in parallel with heat flow being proportional to the difference in temperature).

The conditions for the validity of the hypothesis (10), and hence SEA, are not fully understood yet. However, much research and experience have given guidelines. Some of these, being relevant for the present discussion, are:

(1) Each substructure must have resonances within the considered frequency band since SEA describes resonant and free wave motion.
(2) The modal overlap factor, i.e., the ratio of resonance bandwidth to the average frequency spacing of resonances, must not be too small. For reverberant systems, coupling power is substantial only when modes in connected systems have roughly the same frequency. The probability of resonant interaction may be limited when the modal overlap is small and hence there may be large deviations between the SEA expectation of coupling power and the actual value for a particular structure.
(3) A structure suitable for SEA is irregular and randomly excited because "the essential condition is incoherence between different components of response — either the modal response or, in ray theoretical formulations, components that have travelled different paths to the same point" [7].
(4) Coupling must not be too strong. It is believed that coupling is weak if the modal behaviour of a substructure is not much altered when it is connected to the rest of the structure [10].

3.2. TWO-ELEMENT SEA MODEL OF THE PLATE – OSCILLATOR SYSTEM

The first step in an SEA is to subdivide the structure into 'elements'. For the structure in Figure 1, the apparent elements are the plate and the oscillator. The plate is an appropriate SEA element for frequency bands containing sufficient numbers of resonances. Taking 'sufficient' to be three in 1/3 octave bands, then $3 = n\Delta\omega \approx n\omega/4 = N/4$, where $\Delta\omega$ is the frequency bandwidth and N is the mode count. Thus, the statistical approach is approximately valid above the 10th or, say, 15th resonance in the plate.

The input power from the point force to the plate is given by [3, Section IV.4.c]

$$P_{in} = |F|^2 Y_c, \quad Y_c = \frac{\pi n_p}{2M_p}, \tag{11}$$

where Y_c is the characteristic point mobility, i.e., the one applying for an infinite plate. Equation (11) is in reference [3] derived as a frequency average and upon the assumption of a random force position. It can equally be derived from the expected value of the input mobility of a plate with random frequencies and mode shapes [11].

The oscillator is a proper SEA element only for the frequency band containing its resonance. This is where the largest motion of the oscillator is expected so the SEA is of practical interest. For this frequency band, the modal density of the oscillator is given by

$$n_o = 1/\Delta\omega, \tag{12}$$

where the frequency band $\Delta\omega$ must be wide enough to contain most of the energy of the oscillator resonance, but it should not be much wider than this.

The oscillator is undamped, so there is no need to evaluate the conductivity. Consequently, applying equation (8) for the plate and the oscillator results in

$$P_{in} = \eta\omega E_p, \quad E_p/n_p = E_o/n_o, \tag{13}$$

where E_p and E_o are the total energies in the plate and oscillator, respectively.

For resonant motion the kinetic and strain energies are on average equal and thus

$$E_p = M_p w^2, \qquad E_o = M_o v^2. \tag{14}$$

Consequently, the m.s. vibration velocities of the plate and the oscillator are given by

$$w^2 = \frac{P_{in}}{\eta\omega M_p} = \frac{\pi\,|F|^2 n_p}{2\eta\omega M_p^2}, \tag{15}$$

$$\frac{v^2}{w^2} = \frac{M_p}{n_p \Delta\omega \mathrm{M}_o}. \tag{16}$$

3.2.1. *SEA coupling factor*

The coupling loss factor is not needed for the considered structure, yet it is for more general structures and also if the oscillator has internal damping. To find the coupling loss factor, consider free motion of the oscillator connected to the plate. The plate dimensions as well as the oscillator position are assumed to be random, so the characteristic mobility applies for describing the expected plate displacement that is induced by the spring force. The equations describing the oscillator motion are

$$-\omega^2 M_o V = f, \quad f = K_o(W_a - V), \quad W_a = -fY_c/\mathrm{i}\omega, \tag{17}$$

where W_a is the plate displacement at the attachment point and the rest of the notation is as in Section 2. The free motion of the oscillator is thus given by

$$\left(\frac{\omega_o^2}{1 + (\omega_o^2 \mathrm{M}_o \mathrm{Y}_c/\omega)^2}\left(1 + \mathrm{i}\,\frac{\omega_o^2 M_o Y_c}{\omega}\right) - \omega^2\right)V = 0. \tag{18}$$

This is the equation of free motion for a damped oscillator, where in this case the 'damping' is caused by transfer of energy from the oscillator to the plate. Upon this basis, for the frequency band containing the oscillator frequency, the coupling loss factor is identified

$$\eta^{op}_{coup} = \omega_o M_o Y_c = \sqrt{K_o M_o} Y_c. \tag{19}$$

It is seen that the coupling loss factor is very large if the oscillator is heavy and has a high resonance frequency while the plate is mobile. When the coupling loss factor is a substantial fraction of unity, or larger, it can hardly be correct to consider the oscillator as a separate energy containing entity. In such a situation, it seems more appropriate to describe the structure in Figure 1 by only one SEA element.

3.3. ONE-ELEMENT SEA MODEL

The SEA model derived above provides estimates only of the oscillator vibration for the frequency band containing its resonance. For other frequencies the oscillator is not an SEA element and must be treated as a part of the plate element. The same applies if the coupling loss factor (19) is not much smaller than unity.

The input power estimate (11) applies with the same degree of accuracy for the one- and the two-element SEA models. Consequently, the spatial average m.s. plate velocity is described by equation (15), since the oscillator has no damping.

To find the oscillator vibration as a function of the average plate vibration, SEA inspired "standard" methods are used [3]. Thus, the plate displacement at the oscillator attachment point is by superposition given by

$$W(x_o, y_o) = W_{a1} + W_{a2}, \tag{20}$$

where W_{a1} is the plate displacement when there is no oscillator and W_{a2} is the additional displacement induced by the oscillator spring force. The oscillator position and the plate properties are assumed random, so the characteristic mobility describes the relation between induced plate displacement and spring force. Upon this basis, the oscillator motion is given by

$$-M_o \omega^2 V = f, \quad f = K_o(W_{a1} + W_{a2} - V), \quad W_{a2} = -f Y_c / \mathrm{i}\omega, \tag{21}$$

so that

$$V = W_{a1}/(1 - \omega^2/\omega_o^2 + \mathrm{i}\omega M_o Y_c). \tag{22}$$

It is emphasized that the use of Y_c in place of the point mobility cannot be motivated by frequency averaging the latter, since the equations are frequency dependent. This use is instead, as is standard in SEA, motivated by taking an ensemble average point of view. Thus, equation (22) is believed to apply for the average vibration of an ensemble of similar structures where both the plate's resonance frequencies and the oscillator position are random.

As noted above, the spatial average m.s. plate vibration amplitude is independent of the presence of the oscillator, since it has no damping. Consequently, for random oscillator position, the expected value of W_{a1} is the spatial average r.m.s. plate displacement. Upon this basis, from equation (22), the m.s. vibration velocity of the oscillator is given by

$$v^2 = |\mathrm{i}\omega V|^2 = \frac{w^2}{(1-\omega^2/\omega_o^2)^2 + (\omega M_o Y_c)^2}, \tag{23}$$

where w^2 is given by equation (15).

4. ASYMPTOTIC MODAL ANALYSIS

The AMA is described in references [1,2]; here only some results are recapitulated and discussed. From the AMA, the m.s. vibration velocity of the oscillator is given by [1, equations (27c) and (28b)]

$$v^2 = \frac{|F|^2 Y_c}{\eta_p \omega_c M_p}\, g(\omega_c), \tag{24}$$

$$g(\omega_c) = \frac{\chi^2}{1+\chi^2}\frac{1}{1+\mu}, \tag{25}$$

$$\chi = \frac{2\rho T_p \lambda_p^2}{\pi^2 M_o}, \quad \mu = \frac{M_p/M_o}{(\pi^2 A_p/2\lambda_p^2)^2 + (M_p/M_o)^2},$$

where ω_c is the centre frequency in the frequency band for which the analysis is provided. Y_c is the characteristic point mobility, defined in equation (11). The spatial average m.s. vibration velocity is given by [1, text after equation (27c)]

$$w^2 = |F|^2 Y_c/(\eta_p \omega_c M_p) \tag{26}$$

so that the factor g in equation (25) is the ratio of m.s. oscillator velocity to the spatial average m.s. plate velocity. Equation (26) is equal to the corresponding SEA equation (15).

Here, it is noted that asymptotically for high frequencies, since μ is much smaller than unity at high frequencies, $g(\omega)$ turns to zero as $1/\omega^2$ (6 dB per octave). This is surprising because the spring acts as a vibration isolator and thus, for an ideal spring, at high frequencies we expect a decay of the ratio v^2/w^2 proportional to $1/\omega^4$ (12 dB per octave).

It is also noted that the oscillator frequency, ω_o, is not apparent in equations (24)–(26) and that $g < 1$ so that the oscillator velocity is for all frequencies, and for all systems, smaller than the average plate velocity. This cannot be generally true.

For instance, if the plate mobility is much smaller than the mobility of the mass/spring system, the plate vibration is unaffected by the oscillator. In this case, the plate is a velocity source at the end of the spring and therefore, large vibration amplification is expected at the oscillator, fixed base, resonance frequency.

5. NUMERICAL EXAMPLES

A point force of unit amplitude at all frequencies excites the structure in Figure 1 and the response is calculated by equations (5). The data used are, unless explicitly stated, those found in Table 1. The calculations are made for the 1/3-octave bands 200 Hz to 5 kHz using 16,000 frequency points; allowing for aproximately 4·5 points within the resonance's 3 dB bandwidth. It is assumed both in AMA and in SEA that results, on average, are independent of the boundary conditions and thus for convenience the plate is simply supported, so the natural modes are sinusoidal as in reference [1, (32)]. In the modal summations in equations (4) and (5), all modes with resonance frequencies below 10 kHz are included (roughly 1500).

5.1. ORIGINAL SYSTEM

First, an oscillator frequency of $f_o = 250$ Hz is considered ($f_o = \omega_o/2\pi$). Figure 2 shows in narrowband, the spatial average m.s. vibration velocities of the plate and

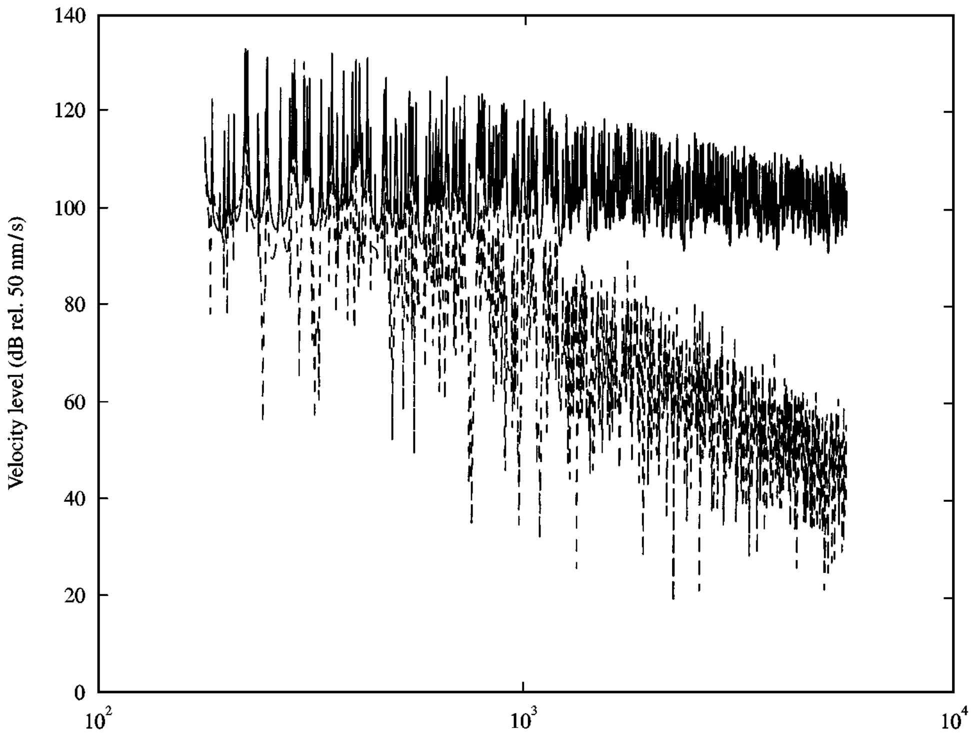

Figure 2. Vibration velocity; ——, spatial average plate velocity level (5a); – – –, oscillator mass velocity level (5b).

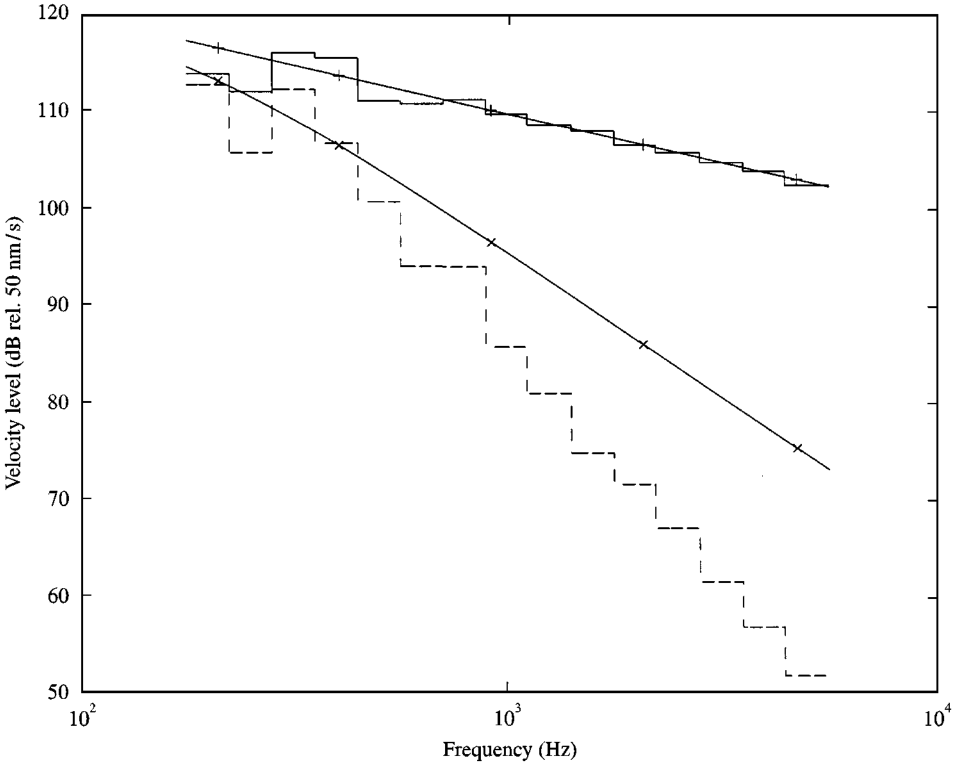

Figure 3. Vibration velocity; Spatial average plate velocity level, ——, exact (5a) 1/3 octave band averaged, – + –, AMA (26); oscillator mass velocity level, – – –, exact (5b) 1/3 octave band averaged, – × –, AMA (24).

oscillator mass calculated with equation (5). Figure 3 shows the same results in 1/3-octave bands as well as the AMA results (24) and (26). Finally, Figure 4 shows in 1/3 octave bands, the ratio of the m.s. oscillator velocity to the spatial average m.s. plate velocity calculated with the exact equations (5), AMA (25), the two-element SEA model (16) and the one-element SEA model (23). For reference, Figure 4 also shows two curves with 6 and 12 dB per octave decay respectively.

As in Figure 3, the AMA prediction (26) [and equally the SEA prediction (15)] of the plate vibration is very good at high frequencies, while at lower frequencies it is only quite good, possibly because of "statistical" fluctuations depending on the exact resonance frequencies and positions for the applied force and the oscillator. The AMA prediction of the oscillator response exhibits the same, presumably stochastic, variation at lower frequencies. However, at higher frequencies, the AMA largely overpredicts the oscillator vibration. Figure 4 confirms this and shows that, for high frequencies, the AMA predicts a 6 dB per octave decrease of the ratio v^2/w^2 instead of a 12 dB decrease, which applies for the true motion.

The two-element SEA model overpredicts the oscillator response by roughly 10 dB. This error is not surprising, as the coupling loss factor is large, $\eta_{coup}^{op} = 1{\cdot}3$.

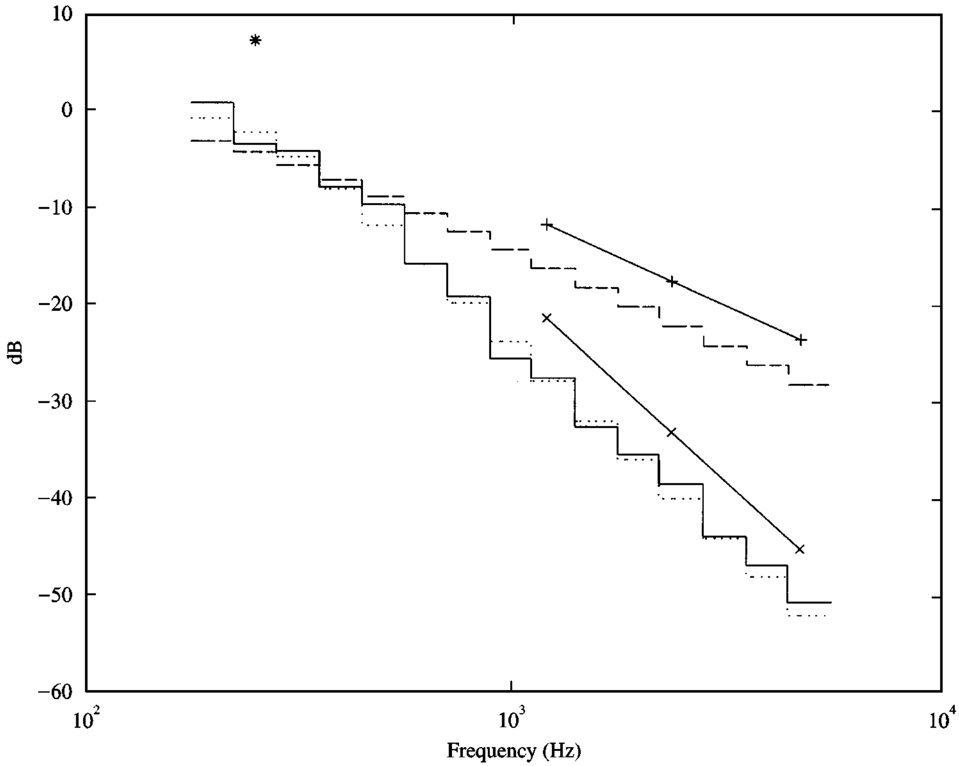

Figure 4. 1/3 octave band averaged ratio of oscillator velocity to spatial average plate velocity, original structure; ——, exact (5); – – –, AMA (25); ··· ···, one-element SEA (23); *, two-element SEA (16); – + –, 6 dB/octave decay; – × –, 12 dB/octave decay.

However, the one-element SEA model gives good results, deviating in all frequency bands by less than 2.5 dB from the exact result.

5.2. WEAKLY COUPLED OSCILLATOR

Two structures with weaker plate–oscillator coupling are investigated. Thus, the mass is varied, the oscillator frequency is $f_o = 1$ kHz and the plate is a 6 mm thick steel plate, $E = 210 \times 10^9$ N/m^2, $\rho = 7800$ kg/m^3—all other data are given in Table 1. The ratio of m.s. oscillator velocity to spatial average m.s. plate velocity is calculated as in the previous section. Figure 5 shows the results for the original mass, $M_o = 110$ g. In this case, $\eta^{op}_{coup} = 0{\cdot}2$, so coupling is not quite weak. Yet, the two-element SEA model is only 3 dB in error compared to the exact result and similarly the one-element SEA model is within $\pm$ 3·5 dB in all frequency bands.

Figure 6 shows the results for a lighter mass, $M_o = 11$ g, in which case $\eta^{op}_{coup} = 0{\cdot}02$. For this structure, both SEA models are only 0·5 dB in error for the band containing the oscillator frequency. The one-element model is, also for this structure, within $\pm$ 3·5 dB in all frequency bands.

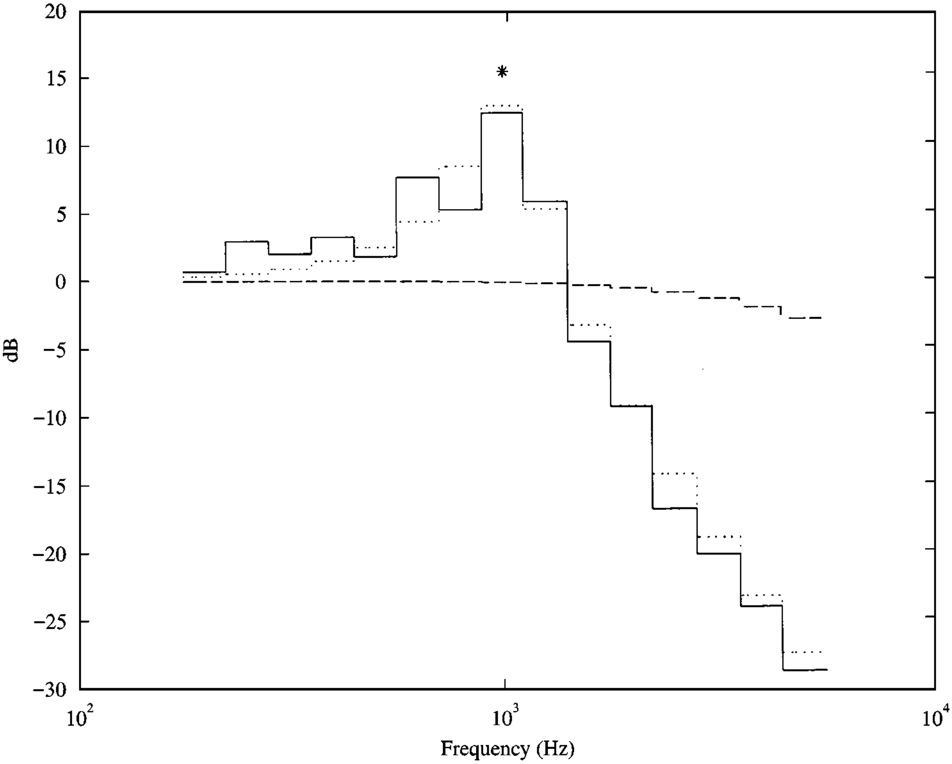

Figure 5. 1/3 octave band averaged ratio of oscillator velocity to spatial average plate velocity, 6 mm steel plate, $M_o = 110$ g. Legends as in Figure 4.

It is noted that for weak coupling, the AMA result (25) is not correct but correct for low frequencies when the mass is almost rigidly connected to the plate.

5.3. ENSEMBLE AVERAGE

To investigate whether the deviations found are due to systematic errors or stochastic fluctuations because of the precise data, ensemble average response is calculated. The ensembles of similar structures are defined by the data used in Sections 5.1 and 5.2. However, the plate dimensions, l_x and l_y, are normal distributed having standard deviations that are 1% of the mean values. Moreover, the force and oscillator positions are assumed uniformly probable over the plate, except that points that are separated by less than 0·12 m are rejected. (0·12 m is slightly less than half a wavelength at the oscillator frequency for both the aluminium and steel plate.) To decrease the computation time, in these calculations, the dissipation loss factor is $\eta = 0{\cdot}01$. Thus, only 1200 frequency points are needed to have 3·5 points within resonance's 3 dB bandwidth. The ensemble averages are calculated by Monte Carlo integration, using 30,000 observations for each of the three cases considered.

Figure 7 shows the ensemble averaged ratio of oscillator velocity to spatial average plate velocity calculated with equation (5) as well as the AMA result (25)

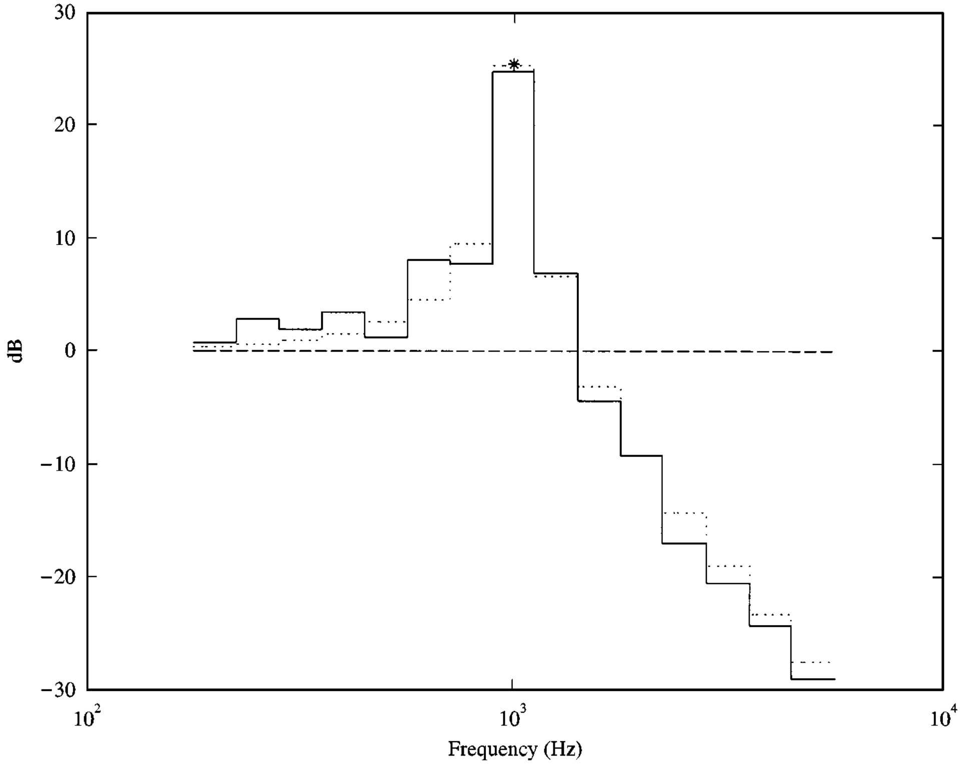

Figure 6. 1/3 octave band averaged ratio of oscillator velocity to spatial average plate velocity, 6 mm steel plate, $M_o = 11$ g. Legends as in Figure 4.

and the one-element SEA result (23). It is seen that the SEA result is in excellent agreement with the ensemble average.

Figure 8 shows the overestimation of the results form the one-element SEA model (23) compared to the ensemble averages found from the exact equations (5). The SEA predicts the ensemble average response accurately for all frequency bands except for the one containing the oscillator frequency. For this band, the SEA underpredicts ensemble average oscillator response by 2·5 dB for the original structure and slightly more than 1 dB for the other two cases considered. Numerical experiments reveal that this error is decreased if the minimum separation between applied force and oscillator is increased and damping is decreased. This suggests an explanation for the error. The direct field from the force excitation gives a coherent excitation of the oscillator that can not be modelled with SEA.

6. CONCLUSIONS

The vibration response of a point excited, simply supported, thin-walled rectangular plate carrying a concentrated mass/spring system is calculated with three methods. Firstly, the exact solution based on modal summation, previously

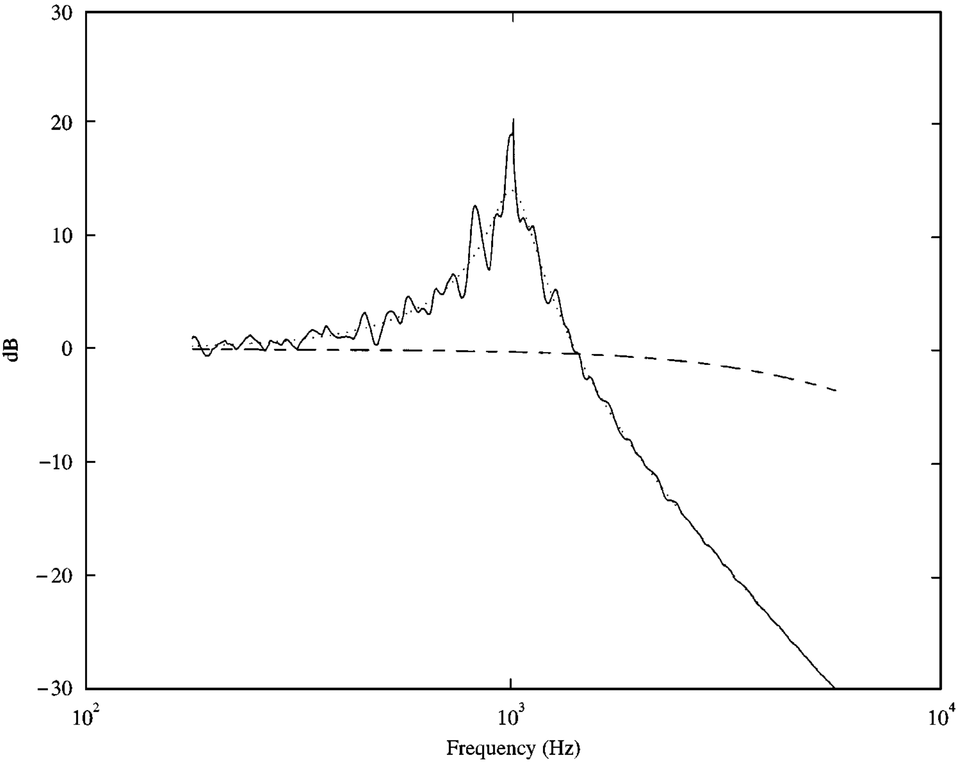

Figure 7. Ratio of oscillator velocity to spatial average plate velocity, 6 mm steel plate, $M_o = 110$ g; ——, exact (5) ensemble averaged; – – –, AMA (25); ⋯ ⋯, one-element SEA (23).

derived in reference [1]. Secondly, the asymptotic modal analysis (AMA) derived in reference [1]. Thirdly, two statistical energy analysis (SEA) models that are derived in the present work.

The first SEA model considers the mass/spring system as one SEA element and the plate as another. This model is valid only for the frequency band containing the oscillator frequency. Moreover, it is valid only if the plate and oscillator are weakly coupled, i.e., if in the frequency band considered, the plate mobility is low compared to the mass mobility. The second SEA model considers the entire structure as one SEA element. Using this model, the oscillator response is found as a function of the plate response using SEA inspired standard methods [3].

The exact solution is used as a reference to assess the accuracy of the two approximate methods. Three different structures are considered; the 2 mm aluminium plate structure investigated in Section 4.2 of reference [1] and two structures with a less mobile, 6 mm, steel plate where the mass either has its original weight of 110 g or is reduced to 11 g.

The AMA is found to be largely in error for frequencies well above the oscillator frequency. For the less mobile steel plate, the AMA is also largely in error at the oscillator frequency. In contrast to this, the two-element SEA model is in error for the aluminium plate structure for which the coupling loss factor has the large value of $\eta^{op}_{coup} = 1{\cdot}3$. However, for the two steel–plate structures, the coupling loss factors

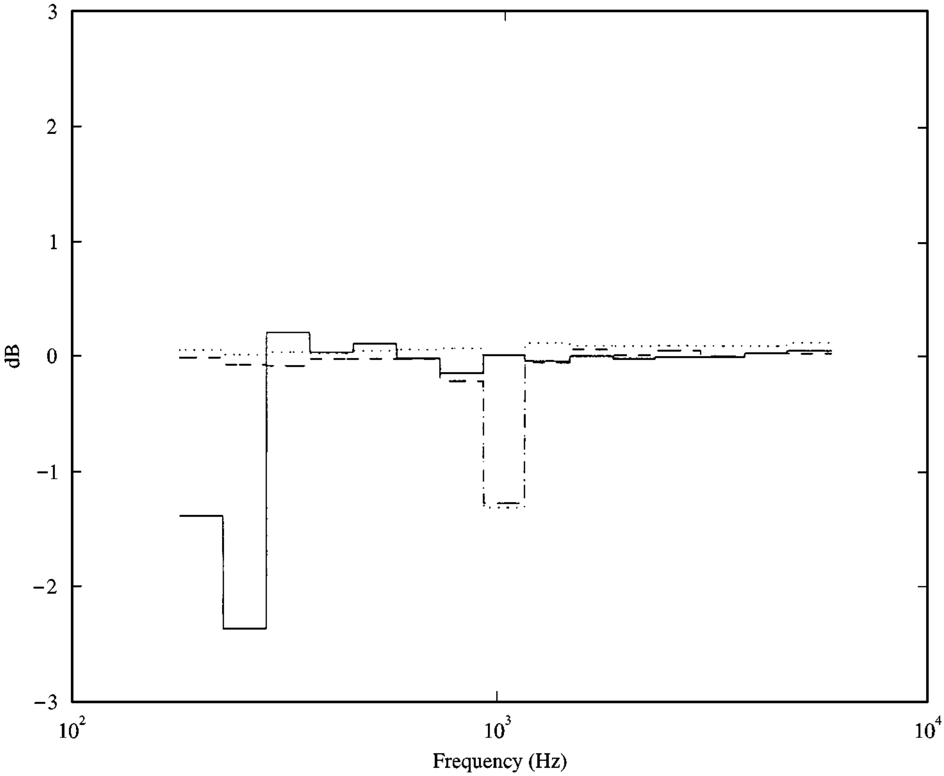

Figure 8. Error in one-element SEA model prediction (23) compared to ensemble averaged exact result (5), '——', original structure; '– – –', 6 mm steel plate, $M_o = 110$ g; '⋯ ⋯', 6 mm steel plate, $M_o = 11$ g.

have the values of $\eta_{coup}^{op} = 0{\cdot}2$ and $\eta_{coup}^{op} = 0{\cdot}02$, respectively, and the errors in the two-element SEA predictions are less than 3 dB. The one-element SEA model predicts the vibrations in all three structures, for all frequency bands, with an error that is less than 3.5 dB.

Ensemble averaged vibration response in the three structures is calculated. The ensembles are defined by that, the plate lengths are normal distributed with a standard deviation that is 1% of the nominal lengths. The one-element SEA model predicts the ensemble average response in all three structures with an error that is less than 6·5 dB in narrow bands and 2·5 dB in third-octave bands.

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AUTHOR'S REPLY

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The authors appreciate the opportunity to comment on the interesting work described in the letter by Dr. Finnveden [1].

First of all, we would like to note the areas of agreement between the results of Dr. Finnveden and ours [2, 3]. Both, the analysis of Dr. Finnveden and ours deduce the same formula and hence the same results for the spatially average mean-square response of the plate *per se* and find that it does not depend on the characteristics of the spring/mass attachment no matter what the mass or stiffness of the attachment. This is a remarkable result which has also been confirmed by experiment for the special case of a very stiff spring in reference [3].

The principal, indeed the only, disagreement between the results of Dr. Finnveden and our own is regarding the response of the spring/mass attachment itself. This issue is a subtle one for both AMA and SEA, for it is clear that the basic premise of many oscillatory modes in the frequency interval of interest (say 1/3 octave) cannot be satisfied by the single-degree-of-freedom spring/mass oscillator *per se*. The plate itself, of course, can and usually does have several modes in the frequency interval of interest.

It is for this reason, perhaps, that our two approaches diverge. Dr. Finnveden in fact derives two distinct SEA models to describe the spring/mass response. In the